CHAPTER 1

Shifting Students' Beliefs about Competence by Integrating Mathematics Strengths into Tasks and Participation Norms

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Sarah Erickson's eighth graders are brilliant, funny, energetic, generous young people who have many negative ideas about mathematics and do not yet believe that they are capable of success. Erickson was frustrated early in the year, because students would rarely engage with tasks or share ideas with one another, two cornerstones of her classroom community. Erickson knew that her students were all very competent. She firmly believed that they came with a lot of prior knowledge and experiences that would support their new learning, but the students just did not believe her. They did not identify as math learners . . . yet.

This particular class was an intervention course for eighth graders. Erickson was supposed to "remediate" students who were tracked into the class because of their low standardized test scores. These young people were mostly immigrants and students of color who were bused to school each day from poverty-stricken neighborhoods. In addition to this class with Erickson, the students attended a "regular" eighth-grade math class on alternate days. The "regular" math class was also tracked by test scores, and this group was placed in the lowest track the school offered. The tracking system in Erickson's district created a situation in which her students rarely received access to grade-level math. Instead of teaching eighth-grade curriculum and standards, Erickson was supposed to remediate and focus on raising test scores. Rather than teaching to a test, however, Erickson provided her students with opportunities to learn the same content as the young people who were in the higher-tracked math class. She held them to high expectations for participation and understanding and supported them in many ways to be successful.

Recently, the class had been learning how to solve linear equations. Students used Lab Gear (Picciotto 2017) as part of Erickson's "multidimensional" curriculum (Boaler and Staples 2008), which frames mathematics as a discipline that is broad and complex. Lab Gear affords students an opportunity to navigate their conceptual understandings of the fundamental rules of algebra, which often seem arbitrary when presented in traditional curricula. Lab Gear also supports powerful math practices such as connections and communication, thereby promoting both access and intellectual rigor for all students.

Erickson is very attuned to how her students are currently thinking about "solving for x." She recognizes their strengths and how they still need one another. Erickson explains, "We're not quite where we need to be yet. The students need more opportunities to hear others' explanations and justifications as they undo operations and isolate variables. They need experiences recording their processes for solving. I want them to manage negative terms more fluently and be comfortable with the *meaning* of distribution. I want students to explain their reasoning, justify their processes, and move fluidly between the geometric and algebraic representations of expressions and equations."

In addition to teaching new content, Erickson must address the deficit narrative that tracking has imposed on her students for so long. During a recent lesson, she decided to interrupt students while they were trying to work on a task, because they were struggling to talk about the math or to share ideas. However, instead of chiding the students about changing their behaviors, Erickson decided to focus on their strengths. Erickson hoped that by pointing out the many "smart" things students were doing and saying, she could encourage them to re-engage. She knew that the students could move their conversations forward if they would only trust one another and their ideas. Erickson explained, "I need someone to tell me how someone else in their group is smart at solving equations. Be specific!" A person from *each* group raised a hand.

Erickson was excited! Pablo, who typically spends much of class with his head down, immediately called out how both of his team members were mathematically smart: "Tony always has an idea about what to do next," and "Michel is a human calculator. We just ask him what something minus something is, and he can do it in his head."

"Yes! They are listening!" thought Erickson. Reflecting on this moment later, Erickson said, "Pablo was using some of the language I use when I name strengths. I often call Michel a human calculator, and I call Nina 'the reader' because she earned her way out of an intervention class in literacy this year. I love this! I am so impressed that each group came up with something mathematical. We are still struggling to make these changes—to develop a community in which we really own the fact that we are smart and use these smartnesses to learn more math together—but this new way of being math learners is slowly becoming more normal!"

Making a Commitment to Access and Equity

The situation described in the vignette is challenging for Erickson and her students. These young people have likely developed identities as *non*-math learners (Aguirre, Mayfield-Ingram, and Martin 2013) as a result of being isolated in remedial classes for so long by a system that equates competence with test scores. According to Erickson, her students often position themselves as "dumb" or "slow" in the stories they tell (see chapter 4 for more about positioning). They consistently try to convince Erickson that they do not know anything or that the content is too hard. These young people simply do not believe that they can learn important mathematics.

Dispositions and beliefs that students have about themselves as learners can deeply influence their choices about how to engage in classrooms (Nasir 2002). If students cannot see themselves as "math people," they are not likely to risk trying to learn something new. Part of our job as teachers then is to attend to the identities that students are developing and actively cultivate positive math identities through our curriculum and instruction.

We (the authors of this chapter) support positive identity development through strengths-based teaching and learning practices. Rather than focusing only on what students do not yet know or cannot yet do, we notice and name competencies to leverage new learning and help young people recognize how they are mathematically capable. Every child comes to us with prior knowledge, ways of doing mathematics and understanding it as a result of living in the world. However, too often we educators tend to focus on what is missing or what our students do not yet understand. We spend a lot of time fixing mistakes or filling gaps. Noticing and naming students' strengths instead of weaknesses and leveraging strengths for new learning can powerfully shape our classroom communities and affect how students think about themselves and one another as math learners (Aguirre, Mayfield-Ingraham, and Martin 2013; Jilk 2014). A strength-based approach to teaching and learning mathematics promotes the belief that we all know some things and no one of us knows everything. We need one another in order to learn more mathematics, and we are truly better together.

Advancing Access and Equity

We use Complex Instruction (CI) (Cohen and Lotan 1997, 2014) to advance an equity agenda in our classrooms. CI is a strength-based pedagogy that aims to "disrupt typical hierarchies of who is 'smart' and who is not" (Sapon-Shevin 2004, p. 3) in heterogeneous classrooms in a collaborative learning environment. CI aims to develop a "mixed set of expectations for competence," meaning that students recognize that everyone has strengths and everyone has weaknesses. We all have something to offer and we all have something yet to learn.

The following three principles of CI, when enacted simultaneously, support access, participation, and learning:

- **Multiple-ability curriculum** is organized around big math ideas that are connected, of high cognitive demand, require multiple representations, and afford multiple points of entry
- **Participation norms and student roles** describe how students do math together and support small-group autonomy and interdependence
- People are positioned with high or low status and assumed to be more or less competent based on valued status characteristics such as gender, skin color, reading ability, or language. Those who are perceived as more competent are assigned high status and tend to participate and learn more. Those who are perceived as less competent are assigned low status and tend to participate and learn less (Berger, Cohen, and Zelditch 1972). These hierarchies of assumed competence, which are formed in our classrooms, can greatly affect access, participation, and learning (Cohen and Lotan 1997).

CI is a framework for both proactively addressing status and mitigating status issues in the moment (Cohen and Lotan 2014; Featherstone et al. 2011). Let's look at two ways in which Erickson manages status in her classroom communities by

challenging the deficit narrative about her students' competence and attending to their beliefs about what mathematics is, what it means to be mathematically smart, and who can be smart in mathematics. The CI-specific strategies Erickson uses are to develop groupworthy tasks designed around big math ideas that incorporate students' strengths and to create norms for participation that rely on students' strengths and promote interdependence.

Developing Groupworthy Tasks That Incorporate Students' Mathematics Strengths

Groupworthy tasks can be powerful tools for shifting students' mindsets about competence (Cohen and Lotan 1997; Jilk 2014). Groupworthy tasks have highcognitive demand (Stein et al. 2000), "illustrate important mathematical concepts, allow for multiple representations, include problems that draw effectively on the collective resources of a group, and have several possible solution paths" (Horn 2005, p. 219). Tasks that meet these criteria require many different math strengths to be successful, thereby creating opportunities for students to notice how they and their peers are "smart" and how these ways of being smart contribute to their learning.

Cohen and Lotan (2014) and Featherstone and colleagues (2011) have discussed how to design groupworthy tasks. Here we look at a task that Erickson created and used with her students (see fig. 1.1) and explain why the task is groupworthy. We also consider how Erickson incorporated students' math strengths into the task and participation norms in order to affect beliefs about competence. Erickson's goals and learning objectives for the task are outlined in figure 1.2.

Big Math Ideas as Learning Objectives

At first glance, the activity might look quite traditional. What makes it groupworthy and how did its groupworthiness support students to continue developing positive math identities? First, the task is groupworthy because it was designed around big math ideas rather than small, discrete procedures or skills. An expansive set of learning objectives opens up a task and provides space for intellectual messiness. Rather than prescribing how to solve equations or which procedures to use, the objectives Erickson set up require students to rely on prior knowledge and different ways of doing mathematics that make sense to them. These opportunities to relate to subject matter in ways that are connected to their understandings make it much easier for students to adopt new identities as people who are competent with math (Boaler and Greeno 2000).

BIG IDEA: We will write and solve linear equations algebraically and geometrically.

Your task **as a group** is to:

- SHOW what each integer expression *means* on the Lab Gear mat. DRAW THE DIAGRAM.
- SIMPLIFY the expression on the mat.
- Then, SOLVE the equations, finding the value of one x.

Make sure EACH team member can **explain** how you understand the math.

1) -4 + 2	2) -4 - 12	3) 6 - 12
1) $-4 = 2x - 2$	2) $2x + 12 = -4$	3) $12 - 3x = 6$

Check your solutions to the equations above by using substitution. Show your work below and be prepared to explain how you know you're right!

1)	2)	3)

For the problems below, write an equation that represents the problem and then solve. DON'T FORGET TO WRITE AN ANSWER SENTENCE!

4)	While Juan was online, he bought 5 music albums, all at the same price . The tax on his purchase was \$6, and the total was \$61. What was the price of	5)	Erickson swims 4 times a week at the pool. She swims <u>the same number</u> <u>of laps</u> on Monday, Wednesday and Friday plus 15 laps on Saturday. She swims a total of 51	6)	Over the weekend, Jasmine spent 2 hours on an assignment, and she spent <u>equal amounts of</u> <u>time</u> studying for 4 exams for a total of 16 hours.
	each album?		laps each week. How many laps does Erickson swim on Monday?		How much time did Jasmine spend studying for <u>each</u> exam?

Fig. 1.1. Erikson's groupworthy task

Goals

- High cognitive demand will be maintained through the entire task.
- Students will learn important grade-level mathematics.
- All students will have access into and through the task.
- Students will recognize their own and others' math strengths and use these strengths to leverage new learning and support others' understanding.
- Students will rely on each other as intellectual resources.

Learning Objectives

Students will-

- use geometric representations to represent integer expressions;
- simplify integer expressions both geometrically and algebraically;
- solve linear equations geometrically and algebraically and make connections between both representations and explain and justify reasoning;
- check reasoning using evaluation and substitution, understand and explain the meaning of a solution; and
- write algebraic equations from situations.

Fig. 1.2. Erickson's groupworthy task

This task is also groupworthy and attempts to shift students' beliefs about their competence because it is cognitively demanding. Instead of narrowing the learning objectives, an approach that often limits intellectual uncertainty in an effort to boost students' confidence, Erickson does the opposite in this task. She increases the potential for students to *not* know things by requiring so many different ways to be successful throughout the lesson. This pedagogical move makes the task more difficult and simultaneously positions students as more smart and capable. When Erickson offers a challenging learning experience, she is telling students that she believes they can learn meaningful mathematics and trusts that they will.

Consistently providing students with an expanded and more realistic version of the discipline supports our goal of transforming students' beliefs about what it means to be a smart math learner and who can be mathematically successful. "When there are many ways to be successful, many more students are successful" (Boaler and Staples 2008, p. 630). These opportunities offer powerful ways to shift mindsets and promote student success.

What Are My Students' Mathematics Strengths?

In addition to learning objectives designed around big math ideas, this groupworthy task required the strengths of particular students whose math identities needed attention. For example, Erickson knew that Jasmine had a hard time starting a task. She also knew that Jasmine was a rock star with Lab Gear and geometric representations. So Erickson focused on increasing Jasmine's status by placing problems that required geometric representations at the beginning of the task. Erickson trusted that Jasmine would use her strengths to launch herself into the task, demonstrate her competence, and feel more capable as a learner from the very start.

Erickson attended to other students' identity development in similar ways by considering the formatting, organization, and language in the task as well as the scope and placement of each problem. These pedagogical moves might seem small, but they are powerful. When we do not think carefully about theses types of moves, we can inadvertently shut some students down or prevent others from getting involved. Considering when particular math strengths will be needed or making small changes in formatting or language potentially can empower students to engage in ways we might never have expected and come to see themselves as strong mathematics learners. This work is not about tricking young people into participating but explicitly developing authentic opportunities to say, "Look at what you already know and can do. That will help you learn more!"

Attending to strengths is not easy. It requires constant and close attention to students' thinking in the moments when they are making sense of new ideas. Although one successful experience with a groupworthy task is not enough to transform how students think about themselves, consistent implementation of tasks that frame content and learning as multidimensional is an important component of a classroom committed to access and equity.

Figure 1.3 shows some of the ideas about students' strengths that Erickson built into the task presented in figure 1.1. These examples can be used to support your thinking about your own students' strengths: how to name them and incorporate them into tasks.

Connecting Students' Strengths to the Norms for Participation

We also include students' math strengths in the norms we develop for how we want students to participate during a lesson. Among the many participation structures that teachers might use to support more access and equitable

Students will-

use geometric representations to represent integer expressions

- Using Lab Gear to represent linear expressions geometrically is prior knowledge for students. Requiring students to build, draw, and write the expressions and equations provides many students with access into and through the task.
- Tiffany and Jasmine are strong with geometric representations. Requiring geometric representations at the beginning of the task will provide them with access into it.
- Tony and Joy draw excellent diagrams from geometric representations. Their teams will need them as they shift from Lab Gear models to diagrams.

simplify integer expressions both geometrically and algebraically

- Lab Gear provides many students with access to algebraic representations of linear expressions. Some are ready for this move or will draw diagrams to support their understanding.
- Pablo and Tony understand that minus can mean "the opposite." I want them to be challenged with simplifying expressions that have the minus sign in different places in an expression.
- Mary uses color-coding to identify positive and negative integers, and this strength helps her make zero pairs. This is a tool she needs to share with Irene.

solve linear equations geometrically and algebraically and make connections between both representations

- Juan has strong procedural fluency. He will have an opportunity to use this strength when he is faced with order of operations and integer operations.
- Ani recognizes when she doesn't understand something yet. I can count on her to ask Leonardo good questions.
- I put variables on both sides of the equals sign to encourage sense making rather than memorizing.
- Joy predicts the format of a solution and makes sure the *x* is always on the left side of the equation. I want to challenge her with equations that are not easy to solve this way.

check reasoning using evaluation and substitution and understand the meaning of a solution

• Students will solve equations and determine if solutions are correct without asking me.

continued on next page

- Keith understands the meaning of solutions in the form of *a* = *a*, after evaluating equations.
- Joy uses substitution to solve two separate linear equations when she checks her solutions.

write algebraic equations from situations

- Nina translates words into algebra. She will talk out loud, and her group will have access to her thinking.
- Grace recognizes key operator words in context. She underlines the words as she develops an equation. This is a strategy I want her to share with the group.

Fig. 1.3. Learning objectives and students' math strengths

engagement are participation quizzes, shuffle-quizzes, and team challenges (Featherstone et al. 2011; Watanabe and Evans 2015). Each of these structures requires students to have a unique set of strengths to be effective, and therefore many students will need to leverage their mathematical ways of thinking, seeing, and doing for everyone to be successful.

Erickson began the school year with a particular set of classroom norms, so her students were familiar with them when they received this task. In general, Erickson wanted students to think out loud so everyone had access to each other's ideas. She wanted students to work with their papers in the middle of their group's table so ideas were available to everyone. Each student was required to write on her own paper, which encouraged individual accountability, and all team papers needed to match, which supported small-group discussion about the same problem at the same time and ensured that no one was left behind.

Figure 1.4 shows the participation norms that Erickson used to support students' access and interdependence and to attend to students' strengths. The figure also includes Erickson's thinking about her students' strengths. As you read through these ideas, consider the ways in which you might want your students to engage with content and with one another when they are learning. What kinds of participation will support student's access and ongoing engagement? What are your students' participation strengths that you might leverage to support more learning and shift their beliefs about what it means to be a successful math learner?

Students write on their own papers.

- Students are accountable for recording math ideas. The act of recording supports understanding and creates a record of ideas students can revisit.
- I want students to use communication tools to document their thinking on paper. Tony will do this algebraically. Mary will use color-coding.

All team papers are in the middle of the table.

- Papers in the middle of the table provide students with access to each other's ideas, including strengths and mistakes so they have things to discuss.
- Ani relies on this norm a lot, and she constantly reinforces it in her group. When she asks her group to put papers in the middle, they start to talk more.

Think out loud.

- This norm keeps groups focused on the same problem so there is something common to discuss. It is especially useful when students debate ideas and want to ask questions.
- Jasmine will continue thinking out loud and push on her team's ideas until the equation makes sense to her.
- Jose is completely willing to be wrong and will offer his ideas for group vetting. He thinks while he talks and before he writes anything on his paper, so this norm supports his sense making.

All papers match.

- This norm supports students to check in with each other about ideas they are recording and how to write them. The need to check with each other often motivates more discussion, reasoning and justification.
- Shared organization supports access to each other's ideas. Fred and Mary always make sure they can understand their peers' thinking.

All voices are heard.

- This norm supports students to share their strengths and assumes that everyone has something valuable to contribute.
- Jorge does not speak English fluently yet, but if his team expects him to share ideas, then they can access his strengths and Jorge can practice speaking English in a mathematical context.

Fig. 1.4. Norms for participation

The participation norms the Erickson used could work well in any classroom; but we want to explain briefly how we understand them to support our goals of access, interdependence, and beliefs about competence. First, Erickson used knowledge of her students, their needs, and their strengths to create a set of norms that required many different ways of doing mathematics. In addition to expanding the content to which students had access through the learning objectives, Erickson provided an expanded version of what it means to learn mathematics, thereby making it possible for more students to participate and eventually come to see themselves as math people. We might imagine a more narrow set of norms that would eliminate the need for discourse or interaction, especially in a class meant for remediation. However, "a narrow set of mathematical practices within school are problematic, not only because they disenfranchise many students, but because they encourage forms of knowing and ways for working that are inconsistent with the discipline (Boaler and Greeno 2000, p. 191).

In addition to expanding what it means to learn mathematics, the norms used for this task promoted small-group interdependence. The norms necessitated collective math engagement so students would rely on each other as intellectual resources. For example, Erickson wanted Pablo to be confused so he would ask his team for help. She wanted Nina to ask Mary about her color-coding strategy so Nina could learn another way to communicate her ideas. Erickson wanted to create a situation in which students truly needed one another, because they learn more when they do and simultaneously gain access to each other's strengths.

Creating an authentic need for students to communicate and interact might be one of the most powerful practices we use to address deficit mindsets. We have found that when our students genuinely need one another and are supported to stick together through a difficult learning experience, they learn to notice and value the strengths each brings to the process. As a result, our efforts to change how individual students think about themselves become a collective movement that perpetuates a classroom culture in which everyone celebrates: "We are *all* capable. We are *all* smart!" (Jilk 2014, p. 120).

Figure 1.5 shows the groupworthy task again, but annotated with students' names where we expect particular strengths to emerge. This strategy helps us see which students we are attending to and where we can expect interdependence during the lesson.

Reflecting and Taking Action

Erickson must know her students well to successfully adapt or create groupworthy tasks that support them in noticing their mathematical strengths and in coming to understand how capable they are as learners. Erickson also needs to understand

BIG IDEA: We will write and solve linear equations algebraically and geometrically.

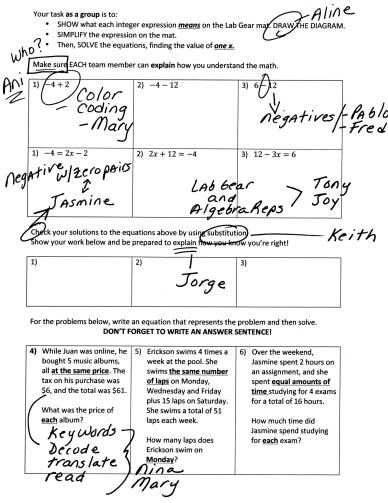


Fig. 1.5. The groupworthy task, annotated

math content broadly, as more than a collection of standards or narrow set of procedures. Finally, Erickson must consider how she wants her students to engage with the content she wants them to learn. What kind of participation with the math and with one another will support their successful entry into the task, their consistent engagement throughout the task, or their interdependence as they work through the task? Considering these ideas often leads to more on-task behavior and therefore more learning for more students, in addition to a shift in beliefs about competence.

We have found it useful to frame our planning and task development with four questions that support our goals for having students learn rigorous mathematics and simultaneously shift their beliefs about who is smart and capable of success:

- 1. What are the big math ideas for this lesson?
 - What are the learning objectives for the task?
 - What do I want my students to be able to do and understand as they work on this task together?
 - How might I expand the mathematical content in this task?
- 2. What are students' mathematical strengths that I can incorporate into this task that are relevant to the learning objectives?
 - What prior knowledge are my students bringing to this task?
 - How are my students smart with these particular big ideas? What can they do? What do they understand? How do they understand?
- 3. What are the norms for participation that will support students' engagement with the task and with each other?
 - What kinds of engagement (spoken, written, listening, pointing, reading) are necessary to be successful with this task?
 - How can students communicate what they learned from engaging with the task (e.g., oral presentation, written summaries, poster presentations)?
- 4. What are students' strengths relative to the participation norms?
 - How do I organize the problems in the task so students can leverage one another's strengths, thereby creating intellectual interdependence?
 - Where might students get stuck and why? What are my students' strengths that will help them to get unstuck?

We have found that attending to students' strengths rather than to their deficits takes deliberate time and attention. We must think carefully about the content in our lessons and give ourselves ample opportunities to notice students' strengths in the moments when they are learning. Sometimes we find it hard to convince ourselves of what it means to be smart in mathematics and to find specific strengths for some of our students. We understand this challenge to be part of our job as math teachers and maybe one of the most important jobs we have. Shifting to strengths is hard work for many reasons. We teachers are trained to pay attention to mistakes and common misconceptions. We want our students to learn mathematics and be successful, so we often help them as much as possible. We are always ready to fix them! Erickson clearly remembers a time not so long ago when she "spent most of the day rescuing kids with the stuff they didn't understand." Her final comments nicely summarize our hopes for how you might use some of these ideas and strategies for helping all young people recognize their mathematical brilliance.

We are already doing so much as teachers, and this is a big shift in how we think about our work and our kids. It's so easy to find students' mistakes. We already know how they're going to mess up, but now I ask myself, "What do my kids need help with that they already know so they can help each other?" They really don't need me. They don't need fixing. They are making sense of ideas and they can do it without me. We can do so much more as teachers by recognizing how great our students already are. They will do WAY more [math] with their smartnesses, because when kids feel smart then they want to keep doing it and share it with everyone.

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